Day 8

Wednesday, December 20, 2023 7:54 AM

Central Limit Theorem:

n is large enough, $\bar{x} \sim N(\mu_{\bar{x}}, \sigma_{\bar{x}})$

$n \geq 30$

$n \to \infty$ $\bar{x} \to N(\mu_{\bar{x}}, \sigma_{\bar{x}})$

μ s.e. $= \frac{\sigma}{\sqrt{n}}$

N: Population size

n: Sample size

$\frac{\sum x_i}{N} = \mu$

Population P.D. σ_x

Sampling distribution of $\bar{x}$ $\sigma_{\bar{x}}$

Population	Sample	Sample mean
N no need	n ✓	NC_n no need
$\mu = \mu_x$	$\bar{x}$ ✓	$E(\bar{x}) = \mu_{\bar{x}} = \mu$
$\sigma = \sigma_x$ unknown	S ✓	$\sigma_{\bar{x}}$: Standard error $= \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}}$ (unknown; $n \geq 30$)

$N = 4$ observations

X_i: $X_1 = 1$, $X_2 = 2$, $X_3 = 3$, $X_4 = 4$

$\mu = \mu_x = 2.5$

$\sigma = \sigma_x = \sqrt{\frac{\sum (X_i - 2.5)^2}{4}}$

Range $= 4 - 1 = 3$

Population P.D.

s.d. of $x = \sigma = \sigma_x$

$\mu = 2.5$

n=2: $X_1 = 1$, $X_3 = 3$, $\bar{x} = 2$, $S = \sqrt{\frac{(1-2)^2 + (3-2)^2}{2-1}}$

n=2: $X_1 = 1$, $X_2 = 2$, $\bar{x} = 1.5$, S

$4C_2 = \frac{4!}{2! \, 2!} = \frac{4 \times 3 \times 2}{4} = 6$

Sample	$\bar{x}$	P
1, 2	1.5	1/6
1, 3	2	1/6
1, 4	2.5	1/6
2, 3	2.5	1/6
2, 4	3	1/6
3, 4	3.5	1/6

$\bar{x}$	P
1.5	1/6
2	1/6
2.5	1/3
3	1/6
3.5	1/6

$\mu_{\bar{x}} = \frac{\sum \bar{x}}{6} = 2.5$

Sampling distribution of $\bar{x}$

n=2

s.d. of $\bar{x} = \sigma_{\bar{x}}$

Range $= 3.5 - 1.5 = 2$

$\mu_{\bar{x}} = \mu$

$\bar{x} = 3.5$ → μ point estimate of μ

$3.5 - 2.5 = 1$ ($\bar{x}$ − μ) Sampling error

$n \geq 30$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}}$

$\bar{x}$

$\mu_{\bar{x}} = \mu$

$y_i = b_0 + b_1 x_{1i} + b_2 x_{2i} + u_i$

y_i: observed value; $\hat{y}_i$: predicted value; u_i: residual (error)

$n \to N$

$n \to \infty$, $\bar{x} \to \mu$

$\mu \leftarrow \bar{x}$ → μ

$\mu \neq \bar{x}$

$E(\bar{x}) = \mu$

Point estimate

$\bar{x} = 80 \to \mu$

Interval estimate

$\mu \in [79, 81]$ w/ 99% Confidence level

90% $\mu \in [75, 85]$

$S = \sqrt{\frac{\sum (X_i - \bar{x})^2}{n-1}} = \sigma$

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Given: μ is to be estimated

n ✓ : $n = 36$

$\bar{x}$ ✓ : $\bar{x} = 80$: point estimate

S ✓ $S = 6$

σ is known/unknown

Confidence level: 95%

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{36}} \approx \frac{S}{\sqrt{36}} = \frac{6}{\sqrt{36}} = 1$

CL:	Z:
95%	±1.96
99%	±2.58
90%	±1.64

95% $\bar{x} \pm Z_{CL} \cdot \frac{S}{\sqrt{n}}$, $n \geq 30$

95% Confidence level

$n \geq 30$ $S \approx \sigma$

$n < 30$ and σ is unknown : $S \neq \sigma$

$n \to \infty$ $t \to Z$

N n

X_1 X_2 X_3

$\sum \frac{X_1 + X_2 + X_3}{3}$ (3 = n)

$\frac{\sum (X_i - \bar{x})^2}{2}$

$E(S^2) = \sigma^2$

$< \frac{n-1}{n} \sigma^2$

$\frac{\sum (X_i - \bar{x})^2}{n}$

b. $E(u_i) = 0$

$y = b_0 + b_1 X_i + u_i$

n-2

$R^2 = 1$