Day 9

Friday, December 22, 2023 8:04 AM

Cases: $n \geq 30$, $\bar{x}=80$, $S=9$ → $80 \pm 1.96 \cdot \frac{S}{\sqrt{n}}$; $n<30$: $S \neq \sigma$

Case 2: $X \sim N(u, \sigma=2.5)$, $n=16<30$, $\bar{x}=80$, $S=4$, σ is known $=2.5$ → $80 \pm 1.96 \cdot \frac{\sigma}{\sqrt{n}}$

Case 3: $X \sim N(u, \sigma)$, $n=16<30$, $\bar{x}=80$, $S=4$, σ is unknown → $80 \pm 2.132 \cdot \frac{S}{\sqrt{n}}$, $d.f. = n-1 = 15$

$n<30$ and σ is unknown : Corrected version of z distribution = t distribution

$\bar{x} \pm t \cdot \frac{S}{\sqrt{n}}$ (d.f., CL)

- $n \to \infty$: $t \to z$
- $n<30$: t
- $n \geq 30$: z

$n<30$ vs $n<30$

$\bar{x} \sim N(u, \frac{\sigma}{\sqrt{n}})$ → $Z_{\bar{x}} \sim Z(0,1)$

$\bar{x} \sim Normal(u, \sigma_{\bar{x}})$ → $Z_{\bar{x}} \sim Z(0,1)$

$-1.96 \cdot \frac{S}{\sqrt{n}}$ $1.96 \cdot \frac{S}{\sqrt{n}}$

$n<30$

$\Pr(a<\bar{x}<b)$

$\frac{\bar{x}_1 - u}{S/\sqrt{n}}$

$\Pr(a<X<b)$ → $\Pr(t_a<t_x<t_b)$

$\Pr(z_a<z_x<z_b)$

Z distribution correction = t distribution

$n \leq 30$, $S \approx \sigma$

P: d.f. = 500, d.f. = 10, t

$\sqrt{n}\frac{\bar{x}-u}{S}$ $\frac{\bar{x}-u}{S/\sqrt{n}} \sim t(0, \text{d.f.})$ d.f. $= n-1$

$\frac{\bar{x}-u}{\sigma/\sqrt{n}} \sim z(0,1)$

One-Sample Test of Hypothesis:

1. H_0 Null : $u=85$ ✓ (unique)

H_1 : H_A : Alternative $u \neq 85$ (research question)

$u_F \leq u_M$

$u_F > u_M$

95%, $n=36$, $\bar{x}=90$, $S=6$

Do Not Reject, 2.5%, 2.5%, $u=85$, 90, 90, $\bar{x}$

Rejection, Rejection, -1.96, 1.96, $\bar{x}=90$, $u=85$ ✗

Critical value

P ↓ ☺

2. Level of significance: $\alpha = \Pr(\text{Type I error}) = 5\%$

Reject a True H_0

3. 1-tail / 2-tail — Z or t test

- Z: $n \geq 30$ or σ is known
- t: $n<30$ and σ is unknown, $S \neq \sigma$

4. Decision Rule: $\bar{x}=90$ → $|Z_{\bar{x}}| \geq Z_{critical}$: Reject

5. Sample data: $\bar{x}=90$, $S=6$, $n=36$

$Z_{\bar{x}} = \frac{\bar{x}-E(\bar{x})}{\sigma_{\bar{x}}} = \frac{90-85}{6/\sqrt{36}} = \frac{5}{1} = 5 > 1.96$: Reject

test statistic

P-Value $= 2 \cdot P(\bar{x} \geq 90)$

$= 2 * (1 - \text{norm.dist}(90, 85, 1, 1)) < 5\%$

H_1: $u \geq 80$ ✗

H_1: $u < 80$ ✓

5%, $\bar{x}$, 80

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test score

5. Inc — Public/Private

$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + u_i$

H_0: $\beta_1 = 0$

H_1: $\beta_1 \neq 0$

$\alpha = 5\%$

Cause: $\Delta X_1 \Rightarrow \Delta Y$

$\Delta X_1 \to \Delta Y$

2.5%, 95%, 2.5%, -1.96, $\beta_1 = 0$, 1.96, b_1, b_1

$\frac{b_1 - E(b_1)}{s.e(b_1)} = \frac{b_1 - 0}{\sigma_{b_1}} > 1.96$