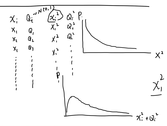


1. $H_0: \sigma_1^2 = \sigma_2^2$ ✗
   $H_1: \sigma_1^2 \neq \sigma_2^2$ ✓
2. $\alpha = 5\%$
3. 2-tail F test
4. $F_{\frac{S_1^2}{S_2^2}} \geq F_{critical}$: Reject

$\underline{X} \sim N(0, 1)$

$\chi^2 \sim \chi^2(1)$

$\sum_{k=1}^{n} X_k^2 \sim \chi^2(k)$



$\underline{X_1^2} + \underline{X_2^2} + \cdots = \sum X_k^2$

$F_i = \frac{S_1^2}{S_2^2} \sim F(df_w, df_b)$

$F_i = \frac{h_1/df_1}{h_2/df_2}$, $h_1 \sim \chi^2(df_1)$, $h_2 \sim \chi^2(df_2)$

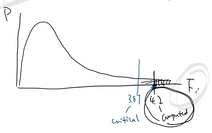
Assumptions:
1) Populations ~ Normal
2) $\sigma_1^2 = \sigma_2^2$

$h_1 = \sum X_i^2 = \sum \left(\frac{X_i - u_1}{\sigma_1}\right)^2$

$h_2 = \sum \left(\frac{X_i - u_2}{\sigma_2}\right)^2$

$F_i = \frac{\sum \left(\frac{X_{1i} - u_1}{\sigma_1}\right)^2 / df_1}{\sum \left(\frac{X_{2i} - u_2}{\sigma_2}\right)^2 / df_2}$ $\sigma_1^2 = \sigma_2^2$ $F_i = \frac{S_1^2}{S_2^2}$

$\frac{\sum (X_{1i} - u_1)^2}{df_1}$ $\frac{\sum (X_{2i} - u_2)^2}{df_2}$



$\frac{\sum (X_i - \bar{X})^2}{d.f.}$

F Distribution:

1. Test $\sigma_1^2 \overset{?}{=} \sigma_2^2$
2. Test: $u_1 = u_2 = u_3 = u_4$

$\sigma_1^2 \overset{?}{=} \sigma_2^2$ ANOVA test

Non-parametric test

χ^2 Distribution

Test: Observed frequency | expected frequency

CLT
- t test :- Assumes population ~ Normal
- Z test : $n \geq 30$ any n
  $n < 30$ need population ~ Normal

$Q_i \sim \chi^2(d.f.)$

$\sum Z_k^2 \sim \chi^2(k)$

$Z_i = \frac{\bar{X} - E(\bar{X})}{\sigma_x/\sqrt{n}}$

$t_i = \frac{\bar{X} - E(\bar{X})}{S_x/\sqrt{n}}$

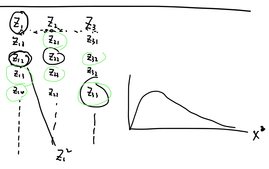
$F_i = \frac{S_1^2}{S_2^2}$

$\chi_i^2 = \sum \frac{(f_o - f_e)^2}{f_e}$

Chi-square statistic

Z_i^2

$\sum \left(\frac{X_i - E(X)}{\sigma_{xi}}\right)^2$



X_i: 1 success, 0 failure $\Rightarrow X_i \sim$ Binomial Distribution

n trials, P_{11} $n \geq 30$

X: # of successes in n trials

$\sum X_i^2 \ldots \sum \left(\frac{X_i - u_i}{\sigma_i}\right)^2$ Normal

$\frac{X - E(X)}{\sigma_x} = \frac{X - np}{\sqrt{npq}} \sim Z(0, 1)$

$\sum \frac{(f_o - f_e)^2}{f_e} \sim \chi^2(d.f.)$

$\left(\frac{X - np}{\sqrt{npq}}\right)^2 \sim \chi^2(1)$

$\frac{(X - np)^2}{npq} = \frac{(X - np)^2}{np} + \frac{(n - X - nq)^2}{nq} \sim \chi^2(1)$

$= \frac{(X - np)^2 (1 - p)}{npq}$

$= \frac{(X - np)^2 (1 - p)}{npq} + \frac{(X - np)^2 \cdot p}{npq}$

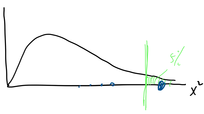
$= \frac{(X - np)^2}{np} + \frac{(X - np)^2}{nq}$

$(np - X)^2 = (n - X - n + np)^2 = (n - X - n(1 - p))^2 = (n - X - nq)^2$

Goodness-of-Fit Test

x_i	f_o	f_e
1	13	20
2	33	20
3	14	20
4	7	20
5	36	20
6	17	20

$\sum \frac{(f_o - f_e)^2}{f_e} \sim \chi^2(5)$



$\frac{(13 - 20)^2}{20} + \frac{(33 - 20)^2}{20} + \cdots + \frac{(17 - 20)^2}{20} = 34.4$

$X \sim$ Binomial (n, p)

$E(X) = np$

$Var(X) = np(1 - p)$

$X \sim$ Binomial (n, q)

$E(X) = nq$

$Var(X) = npq$

$S_1^2 = \frac{\sum (X_i - \bar{X})^2}{d.f.}$

$\sum \left(\frac{(X_i - \bar{X})^2}{\bar{X}}\right) \sim \chi^2$

χ^2

F test

Z^2

$\frac{(X_i - \bar{X})^2}{\sigma_{xi}}$

$\sum \left(\frac{(f_o - f_e)^2}{f_e}\right) = 0$

$f_o \doteq f_e$

error



$y_i = b_0 + b_1 x_1 + b_2 x_2 + x_3$

d.f. ↓ d.f. = 0

$n \geq 50$

$\hat{y}_i = y_i$

$\bar{R}^2$ ↑

X_i

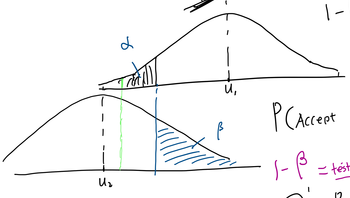
$\bar{X}$

Ordinal

Nominal

$\bar{X} \sim$ Normal

$1 - \beta$



P(Accept False Null) ↑

$1 - \beta$ = test 效能 efficiency

$y_i = b_0 + b_1 b_2 + \ldots$

R^2 $\hat{y} - y$



d.f.N, d.f.D → ∞ F → Norm

$\frac{S_1^2}{S_2^2} \geq 1$

$\frac{S_1}{S_2}$

2% two tail Accept

2% one-tail Reject

1% ~~Reject~~ Accept

5% Reject

Decision Rule ⟹ Conclusion